1.3

18.

W is a subspace of V if and only if 0 is a member of W, and W is closed to the addition and scalar multiplication operations.

when y=0, ax is a member of W for all a in F and all x in W. that means W is closed to the scalar multiplication operation. when a=1, x+y is a member of W which means W is closed the addition operation.

19.



20.

since W is a subspace of V,

scalar multiplications of their members are also in W

and also their additions should be in W

the addition of the first two of the formula should be in W

and then the addition of the result of the previous one and the next one should be in W

then the resulting vector of the formula should be in W

23.

(a)

since both of W have 0, the sum has 0.

so when y=0, x+y=x, and when x=0, x+y=y, which means both of them are in the sum.

let x, z members of the first subspace, y,w members of the second subspace.

(x+z)+(y+w)=(x+y)+(z+w)

since x+y is in the first subspace, z+w in the second subspace,

the resulting vector is also in the sum.

also regarding the scalar multiplication,

a(x+y)=ax+ay where a is a member of F,

ax is a member of the first subspace, ay of the second subspace.

so the sum is closed in addition and scalar multiplication.

this means the sum is a subspace of V

(b)

a subspace of V should be closed with regard to addition operation.

the sum is just the addition of one member from one set and another member from the other set.

since V contains both sets and closed with regard to addition, the addition result of the sum should be in V.

1.4

11.

when x is the only vector,

the span is ax by the definition.

in 3-space, x is a point. and ax, that is the span of {x}, is the infinite line connecting the origin and x.

this interpretation makes sense when x is not 0. when x is 0, the span remains the point itself.

13.



14.



1.5

9.



10.

x=(1, 0, 0)

y=(0, 1, 0)

z=(1, 1, 0)

x+y-z=0

16.

if there is no nontrivial linear combination that results in zero with members of S, then nontrivial linear combinations with the members of proper subsets cannot result in zero since if this is the case, there exists nontrivial linear combination that results in zero with members of S since the linear combination from the proper subset plus linear combination of the members of S-(the proper subset) with all coefficients as zeros is a valid nontrivial linear combination of S that results in zero. if the subset is itself, this subset is surely linearly independent.

and if each finite subset of S is linearly independent, S is linearly independent since S is one of the finite subset of S.